

# Friedmann-Robertson-Walker Models with Late-Time Acceleration

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## Abstract

In order to account for the observed cosmic acceleration, a modification of the ansatz for the variation of density in Friedman-Robertson-Walker (FRW) models given by Islam is proposed. The modified ansatz leads to an equation of state which corresponds to that of a variable Chaplygin gas, which in the course of evolution reduces to that of a modified generalized Chaplygin gas (MGCG) and a Chaplygin gas (CG), exhibiting late-time acceleration.

We consider the homogeneous and isotropic Robertson-Walker space-time

$$ds^2 = -dt^2 + R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (1)$$

described by its scale factor  $R(t)$  and curvature parameter  $k = 0, \pm 1$ . The universe is assumed to be filled with a distribution of matter represented by the energy-momentum tensor of a perfect fluid given by

$$T_{ij} = (\rho + p) U_i U_j + p g_{ij} \quad (2)$$

where  $\rho$  is the energy density of the cosmic matter and  $p$  is its pressure. The Einstein field equations

$$R_{ij} - \frac{1}{2} R_k^k g_{ij} = -8\pi G T_{ij} \quad (3)$$

for the space-time (1) yield the following two independent equations

$$8\pi G \rho = 3 \frac{\dot{R}^2}{R^2} + 3 \frac{k}{R^2} \quad (4)$$

$$8\pi G p = -2 \frac{\ddot{R}}{R} - \frac{\dot{R}^2}{R^2} - \frac{k}{R^2} \quad (5)$$

from which we obtain the conservation equation

$$\frac{d}{dR}(\rho R^3) = -3pR^2 \quad (6)$$

and the Raychaudhuri equation

$$8\pi G(\rho + 3p) = -6\frac{\ddot{R}}{R} \quad (7)$$

In the past few years, there has been spurt activity in discovering the models of the universe in which the expansion is accelerating, fueled by some self-interacting smooth unclustered fluid with high negative pressure collectively known as dark energy [1]-[3]. These models are mainly motivated by the cosmological observations of the supernovae of type Ia [4]-[7]. Dark energy is also supported by other observations, for example, the anisotropy measurements of the cosmic microwave background radiation [8]-[9] and the observations of the baryon acoustic oscillations [10]-[11].

Dark energy can be represented by a large-scale scalar field  $\phi$  dominated either by potential energy or nearly constant potential energy. Such a matter will also have its energy-stress tensor in the form  $T_{ij}^{DE} = (\rho_\phi + p_\phi)U_i U_j + p_\phi g_{ij}$  and its equation of state in the form  $p_\phi = w_\phi \rho_\phi$ , where  $w_\phi$  is a function of time in general. A large class of scalar field dark energy cosmological models have been proposed in recent years, including cosmological constant  $\Lambda$  for which  $w_\phi$  reduces to the value -1 (potential energy dominated scalar field) [12], quintessence [13]-[16], K-essence [17], tachyon [18]-[19], Phantom [20]-[21], ghost condensate [22]-[23], quintom [24]-[27] and spintessence [28]. Scalar fields are not the only possibility for the dark energy but there are some alternatives also. Cosmic acceleration can also be accounted by invoking inhomogeneity [29], [30]. It can also be carried out by using some perfect fluid but obeying "the exotic" equation of state, the so-called Chaplygin gas [31], [32]. Chaplygin gas (CG) is a peculiar perfect fluid characterized by the equation of state  $p = -\frac{A}{\rho}$  ( $A$  is a positive constant). Chaplygin introduced this equation of state [33] as a suitable mathematical approximation for calculating the lifting force on a wing of an airplane in aerodynamics. The same model was rediscovered later in the same context [34]-[35]. The negative pressure following from the Chaplygin equation of state could also be used for the description of certain effects in deformable solids [36], of stripe states in the context of the quantum Hall effect and of other phenomena. The Chaplygin gas emerges as an effective fluid associated with d-branes [37]-[38] and can also be derived from Born-Infeld type Lagrangians [32], [39]. One of its most remarkable property is that it describes a transition from a decelerated cosmological expansion to

a stage of cosmic acceleration.

Islam [40] has made the ansatz, in which the mass-energy density  $\rho$  in Friedmann-Robertson-Walker (FRW) models is given as a function of  $R$  as

$$\rho = \frac{A}{R^4} (R^2 + b)^{1/2} \quad (8)$$

where  $A$  and  $b$  are positive constants and obtained exact solutions connecting radiation and matter eras for all the three cases of FRW models. For small  $R$ , the function  $\rho$  behaves like  $R^{-4}$ , while for large  $R$ , it behaves like  $R^{-3}$ , (the cases of pure radiation and zero pressure of standard models). All the FRW models based on the ansatz (8) are decelerating models throughout the evolution. In order to meet the observational requirement of an accelerating universe at present, it is a physical necessity to propose a modified law for the variation of matter density in the universe. In this regard we make an attempt with the modification of the ansatz (8) as

$$\rho = \frac{A}{R^4} (R^2 + b + cR^8)^{1/2} \quad (9)$$

where  $c$  is a positive constant. The ansatz (9) can alternatively (for mathematical ease) be written as

$$\rho = \sqrt{\frac{\alpha}{R^8} + \frac{\beta}{R^6} + \gamma} \quad (10)$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  are all positive constants. From the conservation equation (6) for  $\rho = \sqrt{\frac{\alpha}{R^8} + \frac{\beta}{R^6} + \gamma}$ , the expression for pressure  $p$  is obtained as

$$p = \left( \frac{\alpha}{3R^8} - \gamma \right) \frac{1}{\sqrt{\frac{\alpha}{R^8} + \frac{\beta}{R^6} + \gamma}} \quad (11)$$

so that the equation of state is given by

$$p = \left( \frac{\alpha}{3R^8} - \gamma \right) \frac{1}{\rho} \quad (12)$$

which corresponds to a variable Chaplygin gas [41]-[43]. In view of equations (10) and (11), we see that

$$(\rho + 3p) = \frac{\left( 2\frac{\alpha}{R^8} + \frac{\beta}{R^6} - 2\gamma \right)}{\sqrt{\frac{\alpha}{R^8} + \frac{\beta}{R^6} + \gamma}} \quad (13)$$

which is positive for very small values of the scale factor  $R$  indicating  $\ddot{R} < 0$  initially (from the Raychaudhuri equation (7)). The model starts from the big bang with  $\dot{R} \rightarrow \infty$  (from the Friedmann equation (4)).

For small values of  $R$ , after the big bang, that is, so long as  $\frac{\alpha}{R^8} \gg \frac{\beta}{R^6} + \gamma$ , we have  $\rho \approx \frac{\sqrt{\alpha}}{R^4}$  and  $p \approx \frac{1}{3} \frac{\sqrt{\alpha}}{R^4}$ , which corresponds to the initial radiation dominated era ( $p = \frac{1}{3}\rho$ ) of the standard model. For the small values of  $R$  satisfying the condition  $\frac{\alpha}{R^8} + \gamma \gg \frac{\beta}{R^6}$ , we get  $\rho \approx \sqrt{\frac{\alpha}{R^8} + \gamma}$  and  $p \approx \frac{1}{3}\rho - \frac{4}{3}\frac{\gamma}{\rho}$ , which corresponds to a modified generalized Chaplygin gas characterized by an equation of state  $p = A\rho - \frac{B}{\rho^n}$  [45]-[46]. As  $R$  further increases, the term  $\frac{\beta}{R^6}$  starts dominating over the term  $\frac{\alpha}{R^8}$  in the expression for density (10). That is, the expression for the density takes the form

$$\rho \approx \sqrt{\frac{\beta}{R^6} + \gamma} \quad , \quad \left( \frac{\alpha}{R^8} \ll \frac{\beta}{R^6} + \gamma \right) \quad (14)$$

and the equation of state reduces to

$$p \approx -\frac{\gamma}{\rho} \quad (15)$$

which corresponds to a Chaplygin gas [31]. For the values of  $R$  satisfying the condition  $R^6 \ll \frac{\beta}{\gamma}$  the expression for the density (14) approximates to

$$\rho \approx \frac{\sqrt{\beta}}{R^3} \quad (16)$$

which corresponds to a FRW universe dominated by dust like matter ( $p = 0$ ). In view of (14) and (15), it follows that the universe turns from decelerated phase of expansion to one of acceleration as soon as  $R^6 > \frac{\beta}{2\gamma}$ .

For very large values of the cosmological radius  $R$  from equation (14), the expression for the density approximates to

$$\rho \approx \sqrt{\gamma} \quad (17)$$

and from equation of state (15)

$$p \approx -\sqrt{\gamma}. \quad (18)$$

From Eqs. (17) and (18), we have  $(\rho + 3p) = -2\sqrt{\gamma} < 0$  i.e.  $\ddot{R} > 0$  (from the Raychaudhuri equation (7)) leading to an accelerating universe. This corresponds to an empty universe with a

cosmological constant  $\sqrt{\gamma}$  (de Sitter universe). Thus we find that the modified ansatz (10) ultimately leads to an accelerated phase of expansion passing through the different phases of decelerated expansion, in agreement with the observations.

It is difficult to integrate the Friedmann equation (4) for  $\rho = \sqrt{\frac{\alpha}{R^8} + \frac{\beta}{R^6}} + \gamma$ , even in the case  $k = 0$ , to obtain the time variation of the scale factor  $R$ ; However, the solution for  $\rho = \sqrt{\frac{\beta}{R^6}} + \gamma$  has already been obtained by Kamenshchik et al. [31]. Further detailed properties and consequences of Chaplygin gas cosmological models have already been discussed by Gorini et al. [47].

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